

# Comment on "Scaling of the quasiparticle spectrum for $d$ -wave superconductors"

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In a recent Letter Simon and Lee (SL) [1] suggested a scaling law for thermodynamic and kinetic properties of superconductors with lines of gap nodes. For example, the heat capacity as a function of temperature and magnetic field for  $T \ll T_c$  and  $H \ll H_{c2}$  is

$$C(T, H) = aT^2 G(x), \quad x = \alpha T/H^{1/2}, \quad (1)$$

where  $G(x)$  is a dimensionless function of the dimensionless parameter  $x$ . Eq. (1) is in agreement with our calculations [2–4] if the scaling parameter is  $x_{KV} = (H_{c2}/H)^{1/2}(T/T_c)$  (apart from a logarithmic factor). Indeed, according to [2], the function  $G(x) \sim 1 + (1/x^2)$  for large values of  $x$ ; the first term is the bulk heat capacity  $C(T, H) \propto T^2$ , while the second term results in a temperature-independent vortex contribution  $C(T, H) \propto H$ . For small  $x$ , the function  $G(x) \sim 1/x$  which gives [3,4]  $C(T, H) \propto T\sqrt{H}$ . The crossover value between these two regimes is  $x_{KV} \sim 1$ . However, SL have obtained the crossover parameter  $x_{SL} \sim (H_{c2}/H)^{1/2}(T/T_c)\sqrt{E_F/T_c}$ . The difference between our  $x$  and that obtained by SL is thus by the large factor  $\sqrt{E_F/T_c}$ . We discuss the origin of this disagreement.

Let us introduce the anisotropic Fermi momentum  $p_F(\theta)$  which depends on the angle  $\theta$  in the  $a-b$  plane. If  $T \ll T_c$  the quasiparticles which are close to the gap node,  $\theta \ll 1$ , are important. Their spectrum is

$$E(\mathbf{p}) = \sqrt{v_F^2(p - p_F(\theta))^2 + (\Delta')^2\theta^2}, \quad (2)$$

where  $v_F$  is Fermi velocity and  $\Delta'$  is the the angular derivative of the gap,  $\Delta(\theta) \approx \theta\Delta'$ , both are in a vicinity of the node. Eq. (2) was the starting point in [2,4].

In contrast, SL used the linearized spectrum

$$E(\mathbf{p}) = \sqrt{c_{\parallel}^2\delta p_x^2 + c_{\perp}^2p_y^2}, \quad (3)$$

where  $\mathbf{p} = p_y\hat{y} + (p_F + \delta p_x)\hat{x}$ ,  $c_{\parallel} = v_F$  and  $c_{\perp} = \Delta'/p_F$ . This is justified when the nonlinear contributions to

$$\epsilon(\mathbf{p}) - E_F = p_x^2/2m_x + p_y^2/2m_y - E_F \approx v_F\delta p_x + p_y^2/2m_y$$

can be neglected, i.e., when  $p_y^2/2m_y \ll c_{\perp}p_y$  where  $p_y = p_F\theta$ . This requires much stronger restrictions both on the angle,  $\theta \ll T_c/E_F$ , and on the energy and temperature:  $T \ll T_c^2/E_F$ .

At the first glance, one might expect that the temperature of order of  $T_c^2/E_F$  marks the boundary between our scaling and that by SL. However, this is not the case. Our quasiclassical approach is valid down to the temperature at which a discreteness of fermion bound states in the vortex background becomes important. For  $s$ -wave superconductors, the interlevel spacing of core fermions is of order of  $T_c^2/E_F$  [5], thus the quantum limit is reached at  $T \sim T_c^2/E_F$ . In  $d$ -wave superconductors, in a vicinity of the gap nodes, the interlevel distance is smaller; it is determined by a large dimension of the wave function which, for low energies, is limited by

the intervortex distance  $R$  [2,3]. Thus the discreteness of the levels becomes important at lower temperatures,  $T \sim (T_c^2/E_F)(\xi/R) \sim (T_c^2/E_F)\sqrt{B/B_{c2}}$ .

Therefore, one expects two changes of the regime with the crossover parameters as follows: (1) At  $x_{KV} \sim 1$  (i.e., at  $(H_{c2}/H)^{1/2}(T/T_c) \sim 1$ ) the single-vortex contribution to the thermodynamic quantity is comparable with the bulk contribution per one vortex. (2) At  $x_{KV} \sim T_c/E_F$  (ie at  $(H_{c2}/H)^{1/2}(T/T_c)(E_F/T_c) \sim 1$ ) the quasiclassical regime changes to the quantum one. However, there is no change in the regime at the SL scale, i.e., at  $x_{SL} \sim 1$  ( $(H_{c2}/H)^{1/2}(T/T_c)\sqrt{E_F/T_c} \sim 1$ ). This is because the high anisotropy of the conical spectrum in Eq.(3) was not taken properly into account in [1]: the rescaling of coordinates to get an isotropic spectrum of fermions with the average "speed of light"  $c = \sqrt{c_{\parallel}c_{\perp}}$  [1] leads to a high deformation of the potential well produced by vortices.

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